TRANSPORTATION PROBLEM

MŰEGYETEM 1782

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Introduction

Streamlined Simplex Method

Excersises

Basic of the transportation method

- Original applications of the linear programming (LP) method
- Basic problem
  - a company manufactures products at m source location (a_i, $i = 1, \ldots, m$)
  - demand for the product is distributed among n different absorption locations (b_j, $j = 1, \ldots, n$)
- Objective: deliver products from source locations to absorption locations at minimum cost
  - c_{ij}: delivery cost of one unit from source location i to absorption location j
  - $c_{ij}x_{ij}$: delivery cost of x_{ij} units from source location i to absorption location j

Linear Programming Method

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = a_i \ \forall i$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ \forall j$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \ \ and \ a_i \geq 0, b_j \geq 0$$

Integer Solutions Property

- **Theorem:** For transportation problems where every a_i and b_j have an integer value, all the basic variables (allocations) in every basic feasible (BF) solution (including an optimal one) also have integer values

Content

Introduction

Streamlined Simplex Method

Excersises

Content – Streamlined Simplex Method

- Introduction
- Finding the Initial Solution
- Distribution Method

Streamlined Simplex Method

- Set up of the tabular form
- Real life examples $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$
  - not always fulfilled
  - dummy supply or destination has needed to introduce
- There are two steps
  - Finding the initial solution
  - Distribution method

i \ j	1	2	…	j	…	n	
1	c_{11}	c_{12}	…	c_{1j}	…	c_{1n}	a_1
2	c_{21}	c_{22}	…	c_{2j}	…	c_{2n}	a_2
…	…	…	…	…	…	…	…
i	c_{i1}	c_{i2}	…	c_{ij}	…	c_{in}	a_i
…	…	…	…	…	…	…	…
m	c_{m1}	c_{m2}	…	c_{mj}	…	c_{mn}	a_m
	b_1	b_2	…	b_j	…	b_n	

Example problem

c_{ij}	1	2	3	4	5	
1	6	3	5	2	7	200
2	3	7	4	4	1	80
3	5	2	3	1	6	130
4	3	5	2	3	2	90
	30	210	60	80	120	

Content – Streamlined Simplex Method

- Introduction
- Finding the Initial Solution
- Distribution Method

Finding the initial solution

- The goal is not to calculate the best solution, but to calculate an initial solution
- Methods
  - Northwest corner route
  - Minimum Costs Method (Dantzig Method)
  - Vogel Initial Programming Method (The Biggest Difference Method)

Northwest Corner Method

- The maximum possible flow is need to be programmed on the top left corner (cell 11)
  - If $b_1 > a_1$ cell 21 is need to be chosen
  - If $b_1 < a_1$ cell 12 is need to be chosen

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	200
2	0	40	40	0	0	80
3	0	0	20	80	30	130
4	0	0	0	0	90	90
	30	210	60	80	120	

$$30 * 6 + 170 * 3 + 40 * 7 + 40 * 4 + 20 * 3 + 80 * 1 + 30 * 6 + 90 * 2 = 1630$$

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	0	0	0	0	0	200
2	0	0	0	0	0	80
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	30	210	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	0	0	0	0	170
2	0	0	0	0	0	80
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	210	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	0	0	0	0	80
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	40	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	0	0	0	40
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	0	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	0	20	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	0	0	110
4	0	0	0	0	0	90
	0	0	0	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	80	0	30
4	0	0	0	0	0	90
	0	0	0	0	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	80	30	0
4	0	0	0	0	0	90
	0	0	0	0	90	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	80	30	0
4	0	0	0	0	90	0
	0	0	0	0	0	

Minimum Costs Method (Dantzig Method)

- Task: program the maximum possible flows on source-absorption location relations, where costs are minimal
- The order:
  - Red
  - Orange
  - Green
  - Purple

x_{ij}	1	2	3	4	5	
1	30	160	0	0	10	**40** **0**
2	0	0	0	0	80	**0**
3	0	50	0	80	0	**50** **0**
4	0	0	60	0	30	**0**
	0	**160** **0**	**0**	**0**	**40** **10**	

$$30 * 6 + 160 * 3 + 10 * 7 + 80 * 1 + 50 * 2 + 80 * 1 + 60 * 2 + 30 * 2 = 1170$$

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^{6}	0^{3}	0^{5}	0^{2}	0^{7}	200
2	0^{3}	0^{7}	0^{4}	0^{4}	0^{1}	80
3	0^{5}	0^{2}	0^{3}	0^{1}	0^{6}	130
4	0^{3}	0^{5}	0^{2}	0^{3}	0^{2}	90
	30	210	60	80	120	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	0^3	0^5	0^2	0^7	200
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	0^2	0^3	80^1	0^6	50
4	0^3	0^5	0^2	0^3	0^2	90
	30	210	60	0	40	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	0^3	0^5	0^2	0^7	200
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	30	160	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	160^3	0^5	0^2	0^7	40
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	30	0	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	160^3	0^5	0^2	0^7	40
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	30	0	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^{6}	160^{3}	0^{5}	0^{2}	0^{7}	40
2	0^{3}	0^{7}	0^{4}	0^{4}	80^{1}	0
3	0^{5}	50^{2}	0^{3}	80^{1}	0^{6}	0
4	0^{3}	0^{5}	60^{2}	0^{3}	30^{2}	0
	30	0	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	30^6	160^3	0^5	0^2	10^7	0
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	0	0	0	0	0	

Vogel Initial Programming Method

- Differency parameter: difference between costs (c_{ij}) of components having the smallest and the-second-smallest unit cost remained in that row or column
- Define the row or column which can be characterized by the highest differency parameter and then from the selected row/column choose the elements, which have the smallest unit costs

$$200 * 3 + 80 * 1 + 10 * 2 + 40 * 3 + 80 * 1 + 30 * 3 + 20 * 2 + 40 * 2 = 1110$$

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	0^6	0^3	0^5	0^2	0^7	200	1
2	0^3	0^7	0^4	0^4	0^1	80	2
3	0^5	0^2	0^3	0^1	0^6	130	1
4	0^3	0^5	0^2	0^3	0^2	90	0
b_j	30	210	60	80	120		
	0	1	1	1	1		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	0^6	0^3	0^5	0^2	0^7	200	1
2	$-^3$	$-^7$	$-^4$	$-^4$	80^1	0	-
3	0^5	0^2	0^3	0^1	0^6	130	1
4	0^3	0^5	0^2	0^3	0^2	90	0
b_j	30	210	60	80	40		
	2	1	1	1	4		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	0^6	0^3	0^5	0^2	$-^7$	200	1
2	$-^3$	$-^7$	$-^4$	$-^4$	80^1	0	-
3	0^5	0^2	0^3	0^1	$-^6$	130	1
4	0^3	0^5	0^2	0^3	40^2	50	1
	30	210	60	80	0		
	2	1	1	1	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	$-^{6}$	0^{3}	0^{5}	0^{2}	$-^{7}$	200	1
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	0^{1}	$-^{6}$	130	1
4	30^{3}	0^{5}	0^{2}	0^{3}	40^{2}	20	1
b_j	0	210	60	80	0		
	-	1	1	1	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	$-^{6}$	0^{3}	0^{5}	$-^{2}$	$-^{7}$	200	2
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	80^{1}	$-^{6}$	50	1
4	30^{3}	0^{5}	0^{2}	$-^{3}$	40^{2}	20	3
	0	210	60	0	0		
	-	1	1	-	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	$-^{6}$	0^{3}	0^{5}	$-^{2}$	$-^{7}$	200	2
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	80^{1}	$-^{6}$	50	1
4	30^{3}	$-^{5}$	20^{2}	$-^{3}$	40^{2}	0	-
	0	210	40	0	0		
	-	1	2	-	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	$-^{6}$	200^{3}	$-^{5}$	$-^{2}$	$-^{7}$	0	-
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	80^{1}	$-^{6}$	50	
4	30^{3}	$-^{5}$	20^{2}	$-^{3}$	40^{2}	0	-
	0	10	40	0	0		
	-			-	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	$-^{6}$	200^{3}	$-^{5}$	$-^{2}$	$-^{7}$	0	-
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	10^{2}	40^{3}	80^{1}	$-^{6}$	0	-
4	30^{3}	$-^{5}$	20^{2}	$-^{3}$	40^{2}	0	-
b_j	0	0	0	0	0		
	-	-	-	-	-		

Content – Streamlined Simplex Method

- Introduction
- Finding the Initial Solution
- **Distribution Method**

Potential System

$$p_{ij} = c_{ij} - v_j + u_i$$

- p_{ij} parameters for the tied elements, are always 0
- Have to choose one row or column arbitrary
- The order:
  - Red
  - Orange
  - Green

x_{ij}	1	2	3	4	5	u_i
1	30^6	160^3	0^5	0^2	10^7	0
2	0^3	0^7	0^4	0^4	80^1	6
3	0^5	50^2	0^3	80^1	0^6	1
4	0^3	0^5	60^2	0^3	30^2	5
v_j	6	3	7	2	7	

- In our example the solution of the Dantzig method is used

Potential System

$$p_{ij} = c_{ij} - v_j + u_i$$

- The numbers: amount of expected change of the sum (Z) value
- Choose the lowest element which is below 0
- If the lowest element is 0, or positive, then the sum will not decrease

	1	2	3	4	5	u_i
1	X	X	-2^5	0^2	X	0
2	3^3	10^7	3^4	8^4	X	6
3	0^5	X	-3^3	X	0^6	1
4	2^3	7^5	X	6^3	X	5
v_j	6	3	7	2	7	

Polygon on the matrix

- A polygon is need to set up on the matrix
- A corner is on the chosen element (now 33)
- Other corners on tied elements
- Sign the corners by ‚+’ and ‚-’ alternately
- Start with ‚+’ and the chosen element

x_{ij}	1	2	3	4	5
1	30	+ 160			10 -
2					80
3		50 -	+	80	
4			60 -		30 +

Current optimum

- The minimum of the ‚-’ signed elements is need to be chosen
- Then this amount is need to add to the ‚+’ signed elements and extract from the ‚-’ signed ones
- The decrease of the Z is the multiplication of the chosen potential number, and the chosen amount

x_{ij}	1	2	3	4	5
1	30	170			
2					80
3		40	10	80	
4			50		40

$$30 * 6 + 170 * 3 + 80 * 1 + 40 * 2 + 20 * 3 + 80 * 1 + 50 * 2 + 40 * 2 = 1140$$

Potential System

- The order:
  - Red
  - Orange
  - Green
  - Purple
  - Blue
  - Brown

x_{ij}	1	2	3	4	5	u_i
1	30^6	170^3	0^5	0^2	0^7	0
2	0^3	0^7	0^4	0^4	80^1	3
3	0^5	40^2	10^3	80^1	0^6	1
4	0^3	0^5	50^2	0^3	40^2	2
v_j	6	3	4	2	4	

Potential System

p_{ij}	1	2	3	4	5	u_i
1	X	X	1^5	0^2	3^7	0
2	0^3	7^7	3^4	5^4	X	3
3	0^5	X	X	X	3^6	1
4	-1^3	4^5	X	3^3	X	2
v_j	6	3	4	2	4	

Polygon on the matrix

x_{ij}	1	2	3	4	5
1	- 30	170 +			
2					80
3		40 -	10 +	80	
4	+		50 -		40

Current optimum

x_{ij}	1	2	3	4	5
1		200			
2					80
3		10	40	80	
4	30		20		40

Potential System

- The order:
  - Red
  - Orange
  - Green
  - Purple
  - Blue
  - Brown

x_{ij}	1	2	3	4	5	u_i
1	0^6	200^3	0^5	0^2	0^7	0
2	0^3	0^7	0^4	0^4	80^1	3
3	0^5	10^2	40^3	80^1	0^6	1
4	30^3	0^5	20^2	0^3	40^2	2
v_j	5	3	4	2	4	

Potential System

p_{ij}	1	2	3	4	5	u_i
1	1^{6}		1^{5}	0^{2}	3^{7}	0
2	1^{3}	7^{7}	3^{4}	5^{4}		3
3	1^{5}				3^{6}	1
4		4^{5}		3^{3}		2
v_j	5	3	4	2	4	

Optimal solution

x_{ij}	1	2	3	4	5
1		200			
2					80
3		10	40	80	
4	30		20		40

Content

Introduction

Streamlined Simplex Method

Excersises

Excersises – Practical Methods

- Dummy supply or destination has needed to introduce
  - Analyse the column and row summarises
  - Introduce the supply OR destination what is needed
  - The dummy supply or destination is an artificial location, which means that the needs are not fulfilled
- When a ij route is not feasible
  - In the matrix usually signed by M
  - In Solver a special condition is needed

Excersises – Solver

- Conditions
  - Row summarisies need to be equal
  - Column summarisies need to be equal
  - Nonnegativity cretiria
  - Special criteria for infeasible routes
    - Nonnegativity: $x_{ij} \geq 0 \; \forall i, j$
    - Special criteria: $x_{ij} \leq 0$ for infeasible routes
    - If $x_{ij} \geq 0$ and $x_{ij} \leq 0$, then $x_{ij} = 0$ for infeasible routes

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