TRANSPORTATION PROBLEM

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Basic of the transportation method

- Original applications of the linear programming (LP) method
- Basic problem
  - a company manufactures products at m source location (a_i, $i = 1, \dots, m$)
  - demand for the product is distributed among n different absorption locations (b_j, $j = 1, \dots, n$)
- Objective: deliver products from source locations to absorption locations at minimum cost
  - c_{ij}: delivery cost of one unit from source location i to absorption location j
  - $c_{ij}x_{ij}$: delivery cost of x_{ij} units from source location i to absorption location j

Linear Programming Method

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = a_i \; \forall i$$

$$\sum_{i=1}^{m} x_{ij} = b_j \; \forall j$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \;\; and \; a_i \geq 0, b_j \geq 0$$

Integer Values

- The Linear Programming cannot guarantee integer results
- However there is a theorem about the integer results
- **Integer solutions property:** For transportation problems where every a_i and b_j have an integer value, all the basic variables (allocations) in *every* basic feasible (BF) solution (including an optimal one) also have *integer* values.

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Streamlined Simplex Method

Excersises

Content – Streamlined Simplex Method

Introduction

Finding the Initial Solution

Distribution Method

Streamlined Simplex Method

- Set up of the tabular form
- Real life examples $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$
  - not always fulfilled
  - dummy supply or destination has needed to introduce
- There are two steps
  - Finding the initial solution
  - Distribution method

i \ j	1	2	…	j	…	n	
1	c_{11}	c_{12}	…	c_{1j}	…	c_{1n}	a_1
2	c_{21}	c_{22}	…	c_{2j}	…	c_{2n}	a_2
…	…	…	…	…	…	…	…
i	c_{i1}	c_{i2}	…	c_{ij}	…	c_{in}	a_i
…	…	…	…	…	…	…	…
m	c_{m1}	c_{m2}	…	c_{mj}	…	c_{mn}	a_m
	b_1	b_2	…	b_j	…	b_n	

Example problem

c_{ij}	1	2	3	4	5	
1	6	3	5	2	7	200
2	3	7	4	4	1	80
3	5	2	3	1	6	130
4	3	5	2	3	2	90
	30	210	60	80	120	

Content – Streamlined Simplex Method

Introduction

Finding the Initial Solution

Distribution Method

Finding the initial solution

- The goal is not to calculate the best solution, but to calculate an initial solution
- Methods
  - Northwest corner route
  - Minimum Costs Method (Dantzig Method)
  - Vogel Initial Programming Method (The Biggest Difference Method)

Northwest Corner Method

- The maximum possible flow is need to be programmed on the top left corner (cell 11)
  - If $b_1 > a_1$ cell 21 is need to be chosen
  - If $b_1 < a_1$ cell 12 is need to be chosen

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	200
2	0	40	40	0	0	80
3	0	0	20	80	30	130
4	0	0	0	0	90	90
	30	210	60	80	120	

$$30 * 6 + 170 * 3 + 40 * 7 + 40 * 4 + 20 * 3 + 80 * 1 + 30 * 6 + 90 * 2 = 1630$$

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	0	0	0	0	0	200
2	0	0	0	0	0	80
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	30	210	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	0	0	0	0	170
2	0	0	0	0	0	80
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	210	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	0	0	0	0	80
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	40	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	0	0	0	40
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	0	60	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	0	0	0	130
4	0	0	0	0	0	90
	0	0	20	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	0	0	110
4	0	0	0	0	0	90
	0	0	0	80	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	80	0	30
4	0	0	0	0	0	90
	0	0	0	0	120	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	80	30	0
4	0	0	0	0	0	90
	0	0	0	0	90	

Northwest Corner Method

x_{ij}	1	2	3	4	5	
1	30	170	0	0	0	0
2	0	40	40	0	0	0
3	0	0	20	80	30	0
4	0	0	0	0	90	0
	0	0	0	0	0	

Minimum Costs Method (Dantzig Method)

- Task: program the maximum possible flows on source-absorption location relations, where costs are minimal
- The order:
  - Red
  - Orange
  - Green
  - Purple

x_{ij}	1	2	3	4	5	
1	30	160	0	0	10	**40** **0**
2	0	0	0	0	80	**0**
3	0	50	0	80	0	**50** **0**
4	0	0	60	0	30	**0**
	0	**160** **0**	**0**	**0**	**40** **10**	

$$30*6+160*3+10*7+80*1+50*2+80*1+60*2+30*2=1170$$

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	0^3	0^5	0^2	0^7	200
2	0^3	0^7	0^4	0^4	0^1	80
3	0^5	0^2	0^3	0^1	0^6	130
4	0^3	0^5	0^2	0^3	0^2	90
	30	210	60	80	120	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	0^3	0^5	0^2	0^7	200
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	0^2	0^3	80^1	0^6	50
4	0^3	0^5	0^2	0^3	0^2	90
	30	210	60	0	40	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	0^3	0^5	0^2	0^7	200
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	30	160	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	160^3	0^5	0^2	0^7	40
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	30	0	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^6	160^3	0^5	0^2	0^7	40
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	30	0	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	0^{6}	160^{3}	0^{5}	0^{2}	0^{7}	40
2	0^{3}	0^{7}	0^{4}	0^{4}	80^{1}	0
3	0^{5}	50^{2}	0^{3}	80^{1}	0^{6}	0
4	0^{3}	0^{5}	60^{2}	0^{3}	30^{2}	0
	30	0	0	0	10	

Minimum Costs Method (Dantzig Method)

x_{ij}	1	2	3	4	5	
1	30^6	160^3	0^5	0^2	10^7	0
2	0^3	0^7	0^4	0^4	80^1	0
3	0^5	50^2	0^3	80^1	0^6	0
4	0^3	0^5	60^2	0^3	30^2	0
	0	0	0	0	0	

Vogel Initial Programming Method

- Differency parameter: difference between costs (c_{ij}) of components having the smallest and the-second-smallest unit cost remained in that row or column
- Define the row or column which can be characterized by the highest differency parameter and then from the selected row/column choose the elements, which have the smallest unit costs

$$200 * 3 + 80 * 1 + 10 * 2 + 40 * 3 + 80 * 1 + 30 * 3 + 20 * 2 + 40 * 2 = 1110$$

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	0^6	0^3	0^5	0^2	0^7	200	1
2	0^3	0^7	0^4	0^4	0^1	80	2
3	0^5	0^2	0^3	0^1	0^6	130	1
4	0^3	0^5	0^2	0^3	0^2	90	0
b_j	30	210	60	80	120		
	0	1	1	1	1		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	0^6	0^3	0^5	0^2	0^7	200	1
2	$-^3$	$-^7$	$-^4$	$-^4$	80^1	0	-
3	0^5	0^2	0^3	0^1	0^6	130	1
4	0^3	0^5	0^2	0^3	0^2	90	0
b_j	30	210	60	80	40		
	2	1	1	1	4		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	0^{6}	0^{3}	0^{5}	0^{2}	$-^{7}$	200	1
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	0^{5}	0^{2}	0^{3}	0^{1}	$-^{6}$	130	1
4	0^{3}	0^{5}	0^{2}	0^{3}	40^{2}	50	1
	30	210	60	80	0		
	2	1	1	1	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	$-^{6}$	0^{3}	0^{5}	0^{2}	$-^{7}$	200	1
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	0^{1}	$-^{6}$	130	1
4	30^{3}	0^{5}	0^{2}	0^{3}	40^{2}	20	1
b_j	0	210	60	80	0		
	-	1	1	1	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	$-^{6}$	0^{3}	0^{5}	$-^{2}$	$-^{7}$	200	2
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	80^{1}	$-^{6}$	50	1
4	30^{3}	0^{5}	0^{2}	$-^{3}$	40^{2}	20	3
	0	210	60	0	0		
	-	1	1	-	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	$-^{6}$	0^{3}	0^{5}	$-^{2}$	$-^{7}$	200	2
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	80^{1}	$-^{6}$	50	1
4	30^{3}	$-^{5}$	20^{2}	$-^{3}$	40^{2}	0	-
	0	210	40	0	0		
	-	1	2	-	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5		
1	$-^{6}$	200^{3}	$-^{5}$	$-^{2}$	$-^{7}$	0	-
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	0^{2}	0^{3}	80^{1}	$-^{6}$	50	
4	30^{3}	$-^{5}$	20^{2}	$-^{3}$	40^{2}	0	-
	0	10	40	0	0		
	-			-	-		

Vogel Initial Programming Method

x_{ij}	1	2	3	4	5	a_i	
1	$-^{6}$	200^{3}	$-^{5}$	$-^{2}$	$-^{7}$	0	-
2	$-^{3}$	$-^{7}$	$-^{4}$	$-^{4}$	80^{1}	0	-
3	$-^{5}$	10^{2}	40^{3}	80^{1}	$-^{6}$	0	-
4	30^{3}	$-^{5}$	20^{2}	$-^{3}$	40^{2}	0	-
b_j	0	0	0	0	0		
	-	-	-	-	-		

Content – Streamlined Simplex Method

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- Finding the Initial Solution
- Distribution Method

Potential System

$$p_{ij} = c_{ij} - v_j + u_i$$

- p_{ij} parameters for the tied elements, are always 0
- Have to choose one row or column arbitrary
- The order:
  - Red
  - Orange
  - Green

x_{ij}	1	2	3	4	5	u_i
1	30^6	160^3	0^5	0^2	10^7	0
2	0^3	0^7	0^4	0^4	80^1	6
3	0^5	50^2	0^3	80^1	0^6	1
4	0^3	0^5	60^2	0^3	30^2	5
v_j	6	3	7	2	7	

- In our example the solution of the Dantzig method is used

Potential System

$$p_{ij} = c_{ij} - v_j + u_i$$

- The numbers: amount of expected change of the sum (Z) value
- Choose the lowest element which is below 0
- If the lowest element is 0, or positive, then the sum will not decrease

	1	2	3	4	5	u_i
1			-2^{5}	0^{2}		0
2	3^{3}	10^{7}	3^{4}	8^{4}		6
3	0^{5}		-3^{3}		0^{6}	1
4	2^{3}	7^{5}		6^{3}		5
v_j	6	3	7	2	7	

Polygon on the matrix

- A polygon is need to set up on the matrix
- A corner is on the chosen element (now 33)
- Other corners on tied elements
- Sign the corners by ‚+’ and ‚-’ alternately
- Start with ‚+’ and the chosen element

x_{ij}	1	2	3	4	5
1	30	+ 160			10 -
2					80
3		50 -	+	80	
4			60 -		30 +

Current optimum

- The minimum of the ‚-’ signed elements is need to be chosen
- Then this amount is need to add to the ‚+’ signed elements and extract from the ‚-’ signed ones
- The decrease of the Z is the multiplication of the chosen potential number, and the chosen amount

x_{ij}	1	2	3	4	5
1	30	170			
2					80
3		40	10	80	
4			50		40

$$30 * 6 + 170 * 3 + 80 * 1 + 40 * 2 + 20 * 3 + 80 * 1 + 50 * 2 + 40 * 2 = 1140$$

Potential System

- The order:
  - Red
  - Orange
  - Green
  - Purple
  - Blue
  - Brown

x_{ij}	1	2	3	4	5	u_i
1	30^6	170^3	0^5	0^2	0^7	0
2	0^3	0^7	0^4	0^4	80^1	3
3	0^5	40^2	10^3	80^1	0^6	1
4	0^3	0^5	50^2	0^3	40^2	2
v_j	6	3	4	2	4	

Potential System

p_{ij}	1	2	3	4	5	u_i
1			1^5	0^2	3^7	0
2	0^3	7^7	3^4	5^4		3
3	0^5				3^6	1
4	-1^3	4^5		3^3		2
v_j	6	3	4	2	4	

Polygon on the matrix

x_{ij}	1	2	3	4	5
1	- 30	170 +			
2					80
3		40 -	10 +	80	
4	+		50 -		40

Current optimum

x_{ij}	1	2	3	4	5
1		200			
2					80
3		10	40	80	
4	30		20		40

Potential System

- The order:
  - Red
  - Orange
  - Green
  - Purple
  - Blue
  - Brown

x_{ij}	1	2	3	4	5	u_i
1	0^6	200^3	0^5	0^2	0^7	0
2	0^3	0^7	0^4	0^4	80^1	3
3	0^5	10^2	40^3	80^1	0^6	1
4	30^3	0^5	20^2	0^3	40^2	2
v_j	5	3	4	2	4	

Potential System

p_{ij}	1	2	3	4	5	u_i
1	1^6	X	1^5	0^2	3^7	0
2	1^3	7^7	3^4	5^4	X	3
3	1^5	X	X	X	3^6	1
4	X	4^5	X	3^3	X	2
v_j	5	3	4	2	4	

Optimal solution

x_{ij}	1	2	3	4	5
1		200			
2					80
3		10	40	80	
4	30		20		40

Content

Introduction

Streamlined Simplex Method

Excersises

Excersises – Practical Methods

- Dummy supply or destination has needed to introduce
  - Analyse the column and row summarises
  - Introduce the supply OR destination what is needed
  - The dummy supply or destination is an artificial location, which means that the needs are not fulfilled
- When a ij route is not feasible
  - In the matrix usually signed by M
  - In Solver a special condition is needed

Excersises – Solver

- Conditions
  - Row summarisies need to be equal
  - Column summarisies need to be equal
  - Nonnegativity cretiria
  - Special criteria for infeasible routes
    - Nonnegativity: $x_{ij} \geq 0 \; \forall i, j$
    - Special criteria: $x_{ij} \leq 0$ for infeasible routes
    - If $x_{ij} \geq 0$ and $x_{ij} \leq 0$, then $x_{ij} = 0$ for infeasible routes

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