LINEAR PROGRAMMING PROBLEMS

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Content

Linear Programming Problem

Simplex Method

Special Issues

Duality Theory

Linear Programming Problem

- Numerous real problem can be modelled as a linear programming problem
- The most well-known solving method is the simplex method, which was constructed by Dantzig in 1947

Linear Programming Problem

- Definition: A linear programming (LP) problem is an optimisation problem for which has the following properties:
  - We attempt to maximise (or minimise) a linear function of the decision variables (objective function)
  - The values of the decision variables must satisfy a set of constraints. Each constraint must be a linear equation or linear inequality.
  - A sign restriction is associated with each variable (optional)

Linear Programming Problem

- Linear objective function that has to be maximized / minimized:

$$Z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$$

- Considering the constraints below:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n (\leq or \geq) b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n (\leq or \geq) b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n (\leq or \geq) b_m$$

- and

$$x_i \geq 0 \; \forall i \in [1..n]$$

Definitions

- The feasible region for an LP is the set of all points that satisfy all the LP's constraints and sign restrictions
- For a maximization problem, an optimal solution to an LP is a point in the feasible region with the best objective function value
- Similarly, for a minimization problem, an optimal solution is a point in the feasible region with the smallest objective function value

Prototype Example

The WYNDOR GLASS CO. produces high-quality glass products, including windows and glass doors. It has three plants. Aluminium frames and hardware are made in Plant 1, wood frames are made in Plant 2, and Plant 3 produces the glass and assembles the products.

Because of declining earnings, top management has decided to revamp the company's product line. Unprofitable products are being discontinued, releasing production capacity to launch two new products having large sales potential:

Product 1: An 8-foot glass door with aluminium framing

Product 2: A 4 6-foot double-hung wood-framed window

Product 1 requires some of the production capacity in Plants 1 and 3, but none in Plant 2. Product 2 needs only Plants 2 and 3. The marketing division has concluded that the company could sell as much of either product as could be produced by these plants. However, because both products would be competing for the same production capacity in Plant 3, it is not clear which *mix of the two products would be most profitable. Therefore, an* OR team has been formed to study this question.

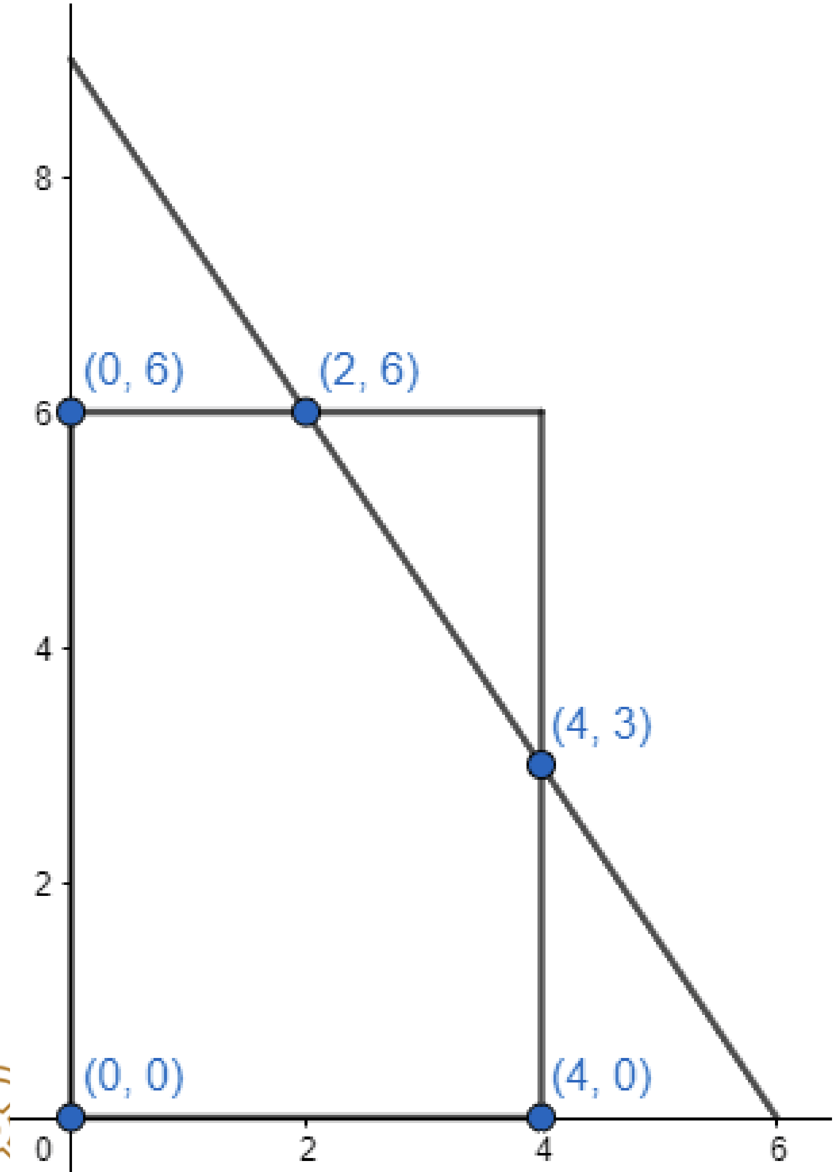
Prototype Example

The OR team began by having discussions with upper management to identify management's objectives for the study. These discussions led to developing the following definition of the problem:

Determine what the *production rates should be for the two products in order to maximize their total profit, subject to the restrictions imposed by the limited production capacities* available in the three plants.

Plant	Production Time per Batch, Hours — Product 1	Production Time per Batch, Hours — Product 2	Production Time Available per Week, Hours
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch	\$3,000	\$5,000	

Geometric Approach



- In case of two decision variable models
- For example

$$\max Z = 3x_1 + 5x_2$$

Subject to

$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$
$$x_i \geq 0 \; \forall i \in [1..n]$$

Simplex Method

- Only takes into consideration the corner point solutions
- In every step, the adjacent corner points are taken into consideration, and a better one is going to be chosen
  - For any linear programming problem with n decision variables, two corner point solutions are adjacent to each other if they share $n-1$ constraint boundaries
- Optimality test: If a corner point solution has no *adjacent* corner point solutions that are *better* (as measured by Z), then it *must* be an *optimal* solution

Simplex Method Algorithm

- Initialisation: finding the initial feasible solution
- Optimality test
- If the current solution is not optimal, then find a better solution among the adjacent corner points

Content

Linear Programming Problem

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Linear Programming Problems (LPPs)

Two forms:

Ad1, General Linear Programming Problems (GLPPs)

conversion (4 steps)

Ad2, Standard Linear Programming Problems (SLPPs)

- *SLPP can be solved with the use of Simplex method*

Conversion of GLPP to standard form (SLPP)- 4 steps

Step 1 – *Write the objective function in the maximization form.*

- If the given objective function is of minimization form then multiply throughout by -1 and write Max z = Min (-z)

Step 2 – *Convert all inequalities as equations*.

- If an equality of '≤' appears then by adding a variable called Slack variable, we can convert it to an equation.
  For example $x_1 + 2x_2 \leq 12$, we can write as $x_1 + 2x_2 + s_1 = 12$.
- If the constraint is of '≥' type, we subtract a variable called Surplus variable and convert it an equation.
  For example $2x_1 + x_2 \geq 15$ can be transformed to $2x_1 + x_2 - s_2 = 15$.

Conversion of GLPP to standard form (SLPP)- 4 steps

Step 3 – *The right side element of each constraint should be made non-negative.*

- For example: $2x_1 + x_2 – s_2 = -15$, must be transformed to $-2x_1 - x_2 + s_2 = 15$ (That is multiplying throughout by -1).

Step 4 – *All variables must have non-negative values*.

- For example:

  $x_1 + x_2 \leq 3$ and $x_1 > 0$; x_2 is unrestricted in sign

  Then x_2 is written as $x_2 = x_2' – x_2''$ where $x_2', x_2'' \geq 0$

  Therefore the inequality takes the form of equation as

  $x_1 + (x_2' – x_2'') + s_1 = 3$

Conversion of GLPP to standard form (SLPP)- Examples

Step 1- Objective function should be in maximization form
Step 2- Convert all inequalities as equations
Step 3- The right side of constraints should be non-negative
Step 4- All variables should be non-negative

EXAMPLE 1:

GLPP:

Minimize $Z = 3x_1 + x_2$

Subject to

$$2x_1 + x_2 \leq 2$$
$$3x_1 + 4x_2 \geq 12$$
$$\text{and } x_1 \geq 0,\ x_2 \geq 0$$

SOLUTION OF EXAMPLE:

SLPP:

Maximize $-Z = -3x_1 - x_2$

Subject to

$$2x_1 + x_2 + s_1 = 2$$
$$3x_1 + 4x_2 - s_2 = 12$$
$$\text{and } x_1 \geq 0,\ x_2 \geq 0,\ s_1 \geq 0,\ s_2 \geq 0$$

Conversion of GLPP to standard form (SLPP)- Examples

Step 1- Objective function should be in maximization form
Step 2- Convert all inequalities as equations
Step 3- The right side of constraints should be non-negative
Step 4- All variables should be non-negative

EXAMPLE 2:

GLPP:

Minimize $Z = x_1 + 2x_2 + 3x_3$

Subject to

$$2x_1 + 3x_2 + 3x_3 \geq -4$$

$$3x_1 + 5x_2 + 2x_3 \leq 7$$

and $x_1, x_2 \geq 0$, and

x_3 is unrestricted in sign

SOLUTION OF EXAMPLE 2:

SLPP:

Maximize $-Z = -x_1 - 2x_2 - 3(x_3' - x_3'')$

Subject to

$$-2x_1 - 3x_2 - 3(x_3' - x_3'') + s_1 = 4$$

$$3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$$

and $x_1 \geq 0, x_2 \geq 0, x'_3 \geq 0,$

$x''_3 \geq 0, s_1 \geq 0, s_2 \geq 0$

Conversion of GLPP to standard form (SLPP)- Examples

Step 1- Objective function should be in maximization form
Step 2- Convert all inequalities as equations
Step 3- The right side of constraints should be non-negative
Step 4- All variables should be non-negative

Convert the following GLPPs to standard form!

EXAMPLE 3:

Maximize $Z = 4x_1 + x_2$

Subject to

$3x_1 + x_2 \leq -3$

$x_1 - x_2 \leq 20$

$-x_1 + 2x_2 \geq -11$

and $x_1 \geq 0, x_2 \geq 0$

EXAMPLE 4:

Minimize $Z = 3x_1 + 2x_2 - x_3$

Subject to

$-x_1 + 3x_2 + 3x_3 \geq -6$

$3x_1 - x_2 + 2x_3 \geq 5$

and $x_1, x_2 \geq 0$, and

x_3 is unrestricted in sign

Conversion of GLPP to standard form (SLPP)- Examples

SOLUTION OF EXAMPLE 3:

SLPP:

Maximize $Z = 4x_1 + x_2$

Subject to

$-3x_1 - x_2 - s_1 = 3$

$x_1 - x_2 + s_2 = 20$

$x_1 - 2x_2 + s_3 = 11$

$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0,$

$s_2 \geq 0, s_3 \geq 0$

SOLUTION OF EXAMPLE 4:

SLPP:

Maximize $-Z = -3x_1 - 2x_2 + (x_3' - x_3'')$

Subject to

$x_1 - 3x_2 - 3(x_3' - x_3'') + s_1 = 6$

$3x_1 - x_2 + 2(x_3' - x_3'') - s_2 = 5$

$x_1 \geq 0, x_2 \geq 0, x'_3 \geq 0, x''_3 \geq 0,$

$s_1 \geq 0, s_2 \geq 0$

Solving a linear programming model using the Simplex method (Example #1)

Maximize $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 \le 4$

$x_1 - x_2 \le 2$

and $x_1, x_2 \ge 0$

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

Solving a linear programming model using the Simplex method

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

Step a,b – *Standard form+Introducing slack/surplus variables*

Maximize $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

and $x_1, x_2 \geq 0$

→

Maximize $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 + s_1 = 4$

$x_1 - x_2 + s_2 = 2$

and $x_1, x_2, s_1, s_2 \geq 0$

Solving a linear programming model using the Simplex method

Step c – *Creating the table (+check optimality)*

Maximize $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 + s_1 = 4$

$x_1 - x_2 + s_2 = 2$

and

$x_1, x_2, s_1, s_2 \geq 0$

x_1	x_2	s_1	s_2	Z	b
1	1	1	0	0	4
1	-1	0	1	0	2
-3	-2	0	0	1	0

Not optimal yet

a. *Standard form*
b. *Introducing slack/surplus variables*
c. ***Creating the table (+check optimality)***
d. *Pivot variables*
e. *Creating a new table*
f. *Checking for optimality*
g. *Identify optimal values*

Solving a linear programming model using the Simplex method

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

Step d – *Pivot variables*

Identifying pivot variable using the table: in the column of the smallest negative value in bottom row; in the row of the smallest non-negative indicator (indicator: divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable)

x_1	x_2	s_1	s_2	Z	b	Indicator
1	1	1	0	0	4	4/1=4
1	-1	0	1	0	2	2/1=**2** ←Smallest non-negat. indicator
-3 Smallest neg. value	-2	0	0	1	0	

Step e – *Creating the new table*

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1)
II. The other values in the column containing the unit value have to become zero
III. During this, the table have to be kept equivalent

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

The old table:

x_1	x_2	s_1	s_2	Z	b
1	1	1	0	0	4
1	-1	0	1	0	2
-3	-2	0	0	1	0

Old pivot column

The new table:

x_1	x_2	s_1	s_2	Z	b
0	2	1	-1	0	2
1	-1	0	1	0	2
0	-5	0	3	1	6

New pivot row

Solving a linear programming model using the Simplex method

Step f – *Checking for optimality*

•**Check optimality using the table**: all values in the last row must contain values greater than or equal to zero

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

x_1	x_2	s_1	s_2	Z	b
0	2	1	-1	0	2
1	-1	0	1	0	2
0	-5	0	3	1	6

Not optimal yet

Solving a linear programming model using the Simplex method

Step d (again) – *Pivot variables*

Identifying pivot variable using the table: in the column of the smallest negative value in bottom row; in the row of the smallest non-negative indicator (indicator: divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable)

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

x_1	x_2	s_1	s_2	Z	b	Indicator
0	2	1	-1	0	2	2/2=**1←Smallest non-negat. indicator**
1	-1	0	1	0	2	2/(-1)= -2 (neglect if negative!!)
0	**-5**	0	3	1	6	
	Smallest neg. value					

Solving a linear programming model using the Simplex method

Step e (again) – *Creating the new table* + **Step f (again)** – *Checking for optimality*

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1)

II. The other values in the column containing the unit value have to become zero

III. During this, the table have to be kept equivalent

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

The old table:

x_1	x_2	s_1	s_2	Z	b
0	2	1	-1	0	2
1	-1	0	1	0	2
0	-5	0	3	1	6

Old pivot column

The new table:

x_1	x_2	s_1	s_2	Z	b
0	1	½	-½	0	1
1	0	½	½	0	3
0	0	5/2	½	1	11

New pivot row

OPTIMALITY REACHED

Solving a linear programming model using the Simplex method

Step g – *Identifying optimal values*

- Basic variables
- Non-basic variables

x_1	x_2	s_1	s_2	Z	b
0	1	½	-½	0	1
1	0	½	½	0	3
0	0	5/2	½	1	11

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

Original task:

Maximize $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 + s_1 = 4$
$x_1 - x_2 + s_2 = 2$
and
$x_1, x_2, s_1, s_2 \geq 0$

Solution:

Max(z) = 11

Subject to

$x_1 = 3$ $s_1 = 0$
$x_2 = 1$ $s_2 = 0$

Solving a linear programming model using the Simplex method (Example #2)

Minimize $-z = -8x_1 - 10x_2 - 7x_3$

Subject to

$$x_1 + 3x_2 + 2x_3 \leq 10$$

$$-x_1 - 5x_2 - x_3 \geq -8$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

Solving a linear programming model using the Simplex method

Step a,b – *Standard form+Introducing slack/surplus variables*

= the 4 steps of converting the GLPP to SLPP!

1. ***Standard form***
2. ***Introducing slack/surplus variables***
3. *Creating the table (+check optimality)*
4. *Pivot variables*
5. *Creating a new table*
6. *Checking for optimality*
7. *Identify optimal values*

Minimize $-z = -8x_1 - 10x_2 - 7x_3$

Subject to

$x_1 + 3x_2 + 2x_3 \leq 10$

$-x_1 - 5x_2 - x_3 \geq -8$

and $x_1, x_2, x_3 \geq 0$

→

Maximize $z = 8x_1 + 10x_2 + 7x_3$

Subject to

$x_1 + 3x_2 + 2x_3 + s_1 = 10$

$x_1 + 5x_2 + x_3 + s_2 = 8$

and $x_1, x_2, x_3, s_1\ s_2 \geq 0$

Solving a linear programming model using the Simplex method

Step c – *Creating the table*

- coefficients corresponding to the linear constraint variables
- coefficients of the objective function (**multiplied by (-1); in the column of 'z': we use 1 in case of max(z), and use -1 in case of max(-z)!**)

Maximize

$z = 8x_1 + 10x_2 + 7x_3$

Subject to

$x_1 + 3x_2 + 2x_3 + s_1 = 10$

$x_1 + 5x_2 + x_3 + s_2 = 8$

and $x_1, x_2, x_3,$

$s_1, s_2 \geq 0$

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

Solving a linear programming model using the Simplex method

Step c – (*+check optimality*)

- **Optimal solution**: the values assigned to the variables in the objective function to give the largest zeta value
- **Check optimality using the table**: all values in the last row must contain values greater than or equal to zero

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

Not optimal yet

Solving a linear programming model using the Simplex method

Step d – *Pivot variables*

- Pivot variable is used to identify, which variable will become the unit value (key factor in conversion)
- **Identifying pivot variable using the table**: in the column of the smallest negative value in bottom row; in the row of the smallest non-negative indicator (indicator: divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable)

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

x1	x2	x3	s1	s2	z	b	Indicator
1	3	2	1	0	0	10	10/3
1	5	1	0	1	0	8	8/5
-8	-10	-7	0	0	1	0	

Smallest neg. value

Smallest non-negative indicator

Solving a linear programming model using the Simplex method

Step e – *Creating the new table*

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1)

II. The other values in the column containing the unit value have to become zero

III. During this, the table have to be kept equivalent

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

➢ *The new table will be used to identify a new possible optimal solution*

Solving a linear programming model using the Simplex method

Step e – *Creating the new table*

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1)

II. The other values in the column containing the unit value have to become zero

III. During this, the table have to be kept equivalent

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

The initial table:

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

Old pivot column

The new table:

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	-3/5	0	26/5
1/5	1	1/5	0	1/5	0	8/5
-6	0	-5	0	2	1	16

New pivot row

Solving a linear programming model using the Simplex method

Step e – *Creating the new table*

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

$$2/5 = (-3) * (1/5) + 1$$

The initial table:

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

Old pivot column

The new table:

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	-3/5	0	26/5
1/5	1	1/5	0	1/5	0	8/5
-6	0	-5	0	2	1	16

New pivot row

Solving a linear programming model using the Simplex method

Step e – *Creating the new table*

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

$$2 = (+10) * (1/5) + 0$$

The initial table:

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

Old pivot column

The new table:

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	-3/5	0	26/5
1/5	1	1/5	0	1/5	0	8/5
-6	0	-5	0	2	1	16

New pivot row

Solving a linear programming model using the Simplex method

Step f – *Checking for optimality*

- **Check optimality using the table**: all values in the last row must contain values greater than or equal to zero

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	-3/5	0	26/5
1/5	1	1/5	0	1/5	0	8/5
-6	0	-5	0	2	1	16

Not optimal yet

Solving a linear programming model using the Simplex method

Step d (again) – *Pivot variables*

- **Identifying pivot variable using the table**: in the column of the smallest negative value in bottom row; in the row of the smallest non-negative indicator (indicator: divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable)

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

x1	x2	x3	s1	s2	z	b	Indicator
2/5	0	7/5	1	−3/5	0	26/5	(26/5) / (2/5) = 13
1/5	1	1/5	0	1/5	0	8/5	(8/5) / (1/5) = 8
-6	0	-5	0	2	1	16	

Smallest neg. value

Smallest non-negative indicator

Solving a linear programming model using the Simplex method

Step e (again) – *Creating the new table*

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1)

II. The other values in the column containing the unit value have to become zero

III. During this, the table have to be kept equivalent

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

The old table:

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	−3/5	0	26/5
1/5	1	1/5	0	1/5	0	8/5
-6	0	-5	0	2	1	16

Old pivot column

The new table:

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
1	5	1	0	1	0	8
0	30	1	0	8	1	64

New pivot row

Solving a linear programming model using the Simplex method

Step f (again) – *Checking for optimality*

- **Check optimality using the table**: all values in the last row must contain values greater than or equal to zero

a. Standard form
b. Introducing slack/surplus variables
c. Creating the table (+check optimality)
d. Pivot variables
e. Creating a new table
f. Checking for optimality
g. Identify optimal values

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
1	5	1	0	1	0	8
0	30	1	0	8	1	64

OPTIMALITY REACHED

Solving a linear programming model using the Simplex method

Step g – *Identifying optimal values*

- **Basic variable**: have a single 1 value in its column and the rest are zeros; the row that contains the 1 value will correspond to the beta value. The beta value will represent the optimal solution for the given variable
- **Non-basic variable**: the remaining variables; the optimal solution of the non-basic variables is zero

a. Standard form
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x1	x2	x3	s1	s2	z	b
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Solving a linear programming model using the Simplex method

Step g – *Identifying optimal values*

- Basic variables
- Non-basic variables

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
1	5	1	0	1	0	8
0	30	1	0	8	1	64

a. Standard form
b. Introducing slack/surplus variables
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Original task:

Maximize $z = 8x_1 + 10x_2 + 7x_3$

Subject to

$x_1 + 3x_2 + 2x_3 + s_1 = 10$

$x_1 + 5x_2 + x_3 + s_2 = 8$

and $x_1, x_2, x_3, s_1, s_2 \geq 0$

Solution:

Max(z) = 64

Subject to

$x_1 = 8$ $s_1 = 2$

$x_2 = 0$ $s_2 = 0$

$x_3 = 0$

Example 2- Solving a linear programming model using the Excel software

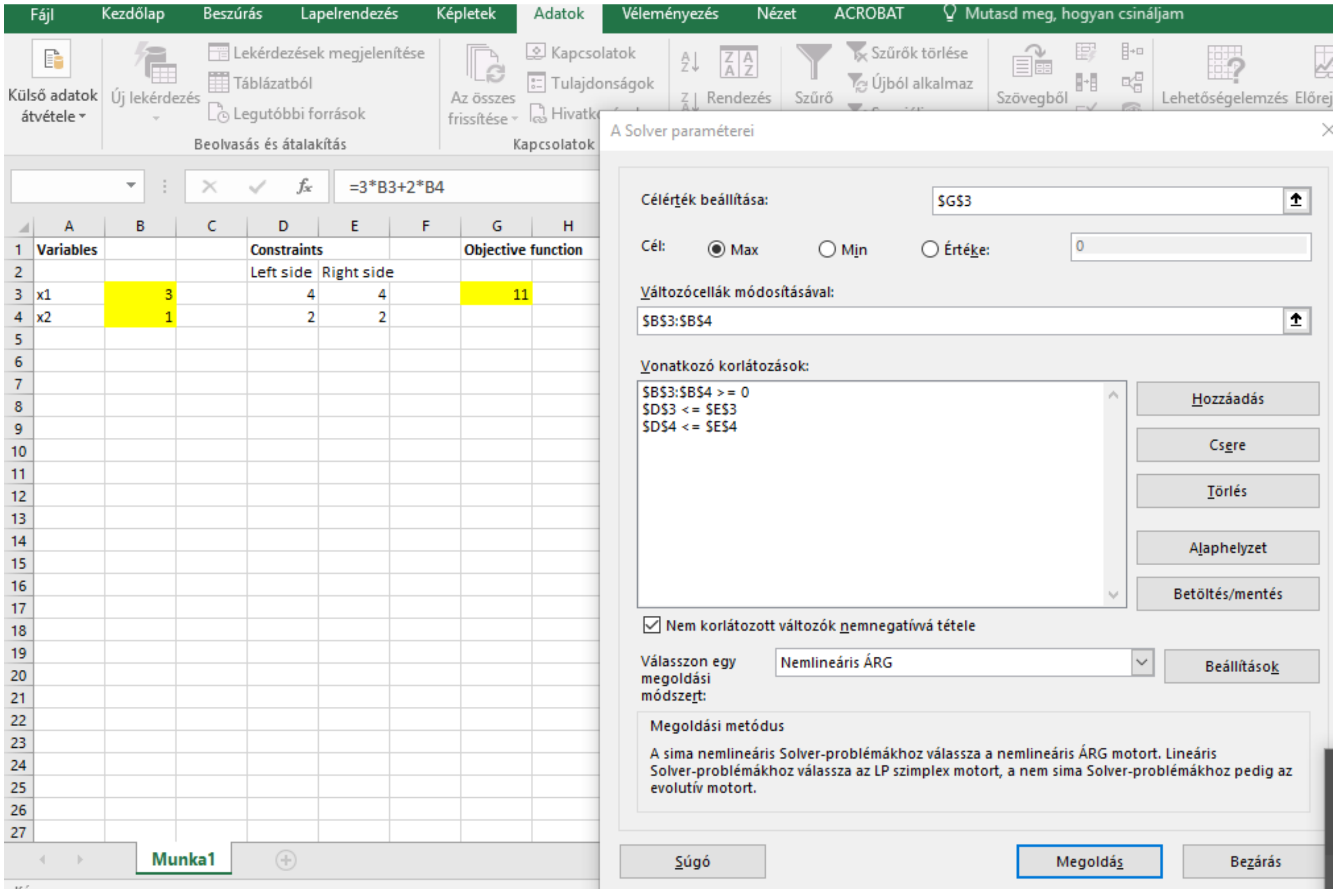
Maximize $Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

and $x_1, x_2 \geq 0$



Example 2- Solving a linear programming model using the MATLAB software

Linear programming solver

Finds the minimum of a problem specified by

$$\min_x f^T x \text{ such that } \begin{cases} A \cdot x \le b, \\ Aeq \cdot x = beq, \\ lb \le x \le ub. \end{cases}$$

f, *x*, *b*, *beq*, *lb*, and *ub* are vectors, and *A* and *Aeq* are matrices.

Syntax:

x= linprog(f,A,b,Aeq,beq,lb,ub)

Finds the minimum of a problem, taking the constraints and bounds into account.

Tutorial:

https://www.mathworks.com/help/optim/ug/linprog.html?requestedDomain=www.mathworks.com#zmw57dd0e65600

Field Name	Entry
`f`	Linear objective function vector `f`
`Aineq`	Matrix for linear inequality constraints
`bineq`	Vector for linear inequality constraints
`Aeq`	Matrix for linear equality constraints
`beq`	Vector for linear equality constraints
`lb`	Vector of lower bounds
`ub`	Vector of upper bounds
`solver`	`'linprog'`

Where:

f: coefficient vector, represents the objective function;

A, b: linear inequality constraints (encodes A*x <= b); Set A=[] and b=[] if no inequ. exists.

Aeq, beq: linear equality constraints (encodes Aeq*x=beq); Aeq=[] and beq=[] if no equ. exists

lb: lower bounds of x (encodes x(i) >= lb(i) for all i);

ub: upper bounds of x (encodes x(i) =< ub(i) for all i).

Example 2- Solving a linear programming model using the MATLAB software

In MATLAB, we need the problem in minimization form, and the constraining inequalities in ≤ form!

Maximize $Z = 3x_1 + 2x_2$

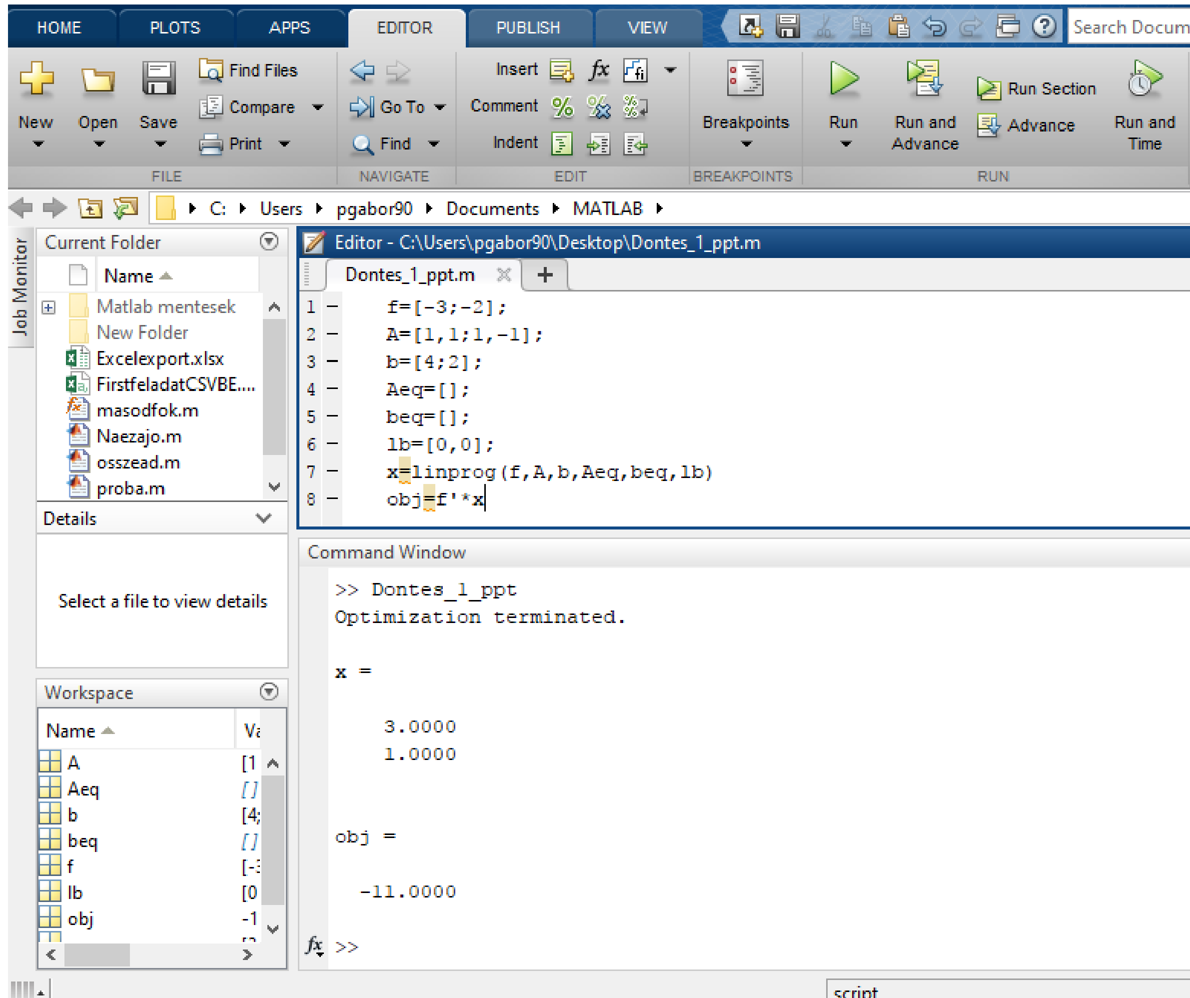
Min $-Z = -3x_1 - 2x_2$

Subject to

$$x_1 + x_2 \le 4$$

$$x_1 - x_2 \le 2$$

$$\text{and } x_1, x_2 \ge 0$$



Example 3- Solve the following LPP!

Solve the following LPP by hand, by Excel and by MATLAB!

Maximize $Z = 5x_1 + 7x_2$

Subject to

$$x_1 + x_2 \leq 4$$

$$-3x_1 + 8x_2 \geq -24$$

$$10x_1 + 7x_2 \leq 35$$

$$\text{and } x_1, x_2 \geq 0$$

a. Standard form
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g. Identify optimal values

Example 3- Solution

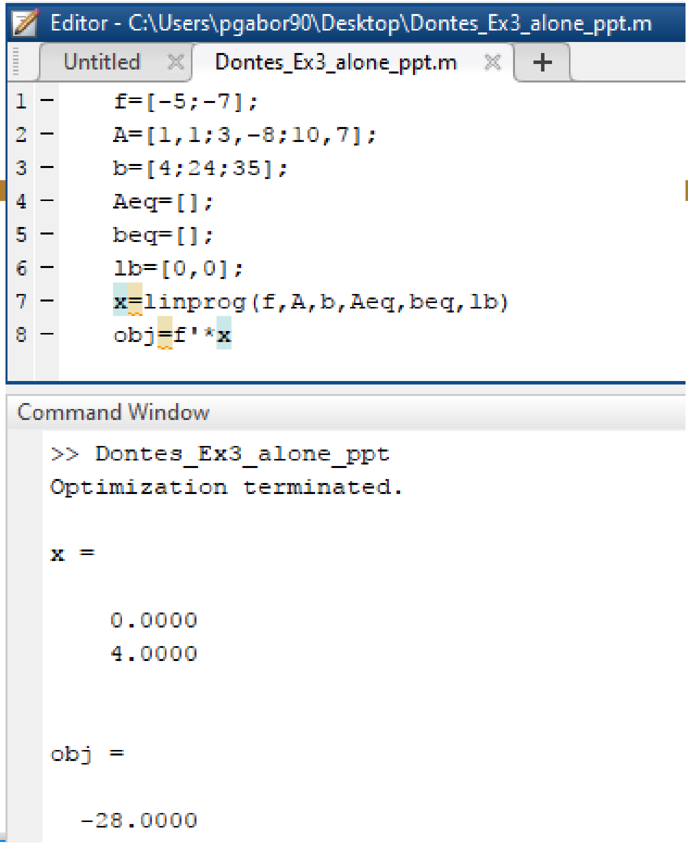
Solution:

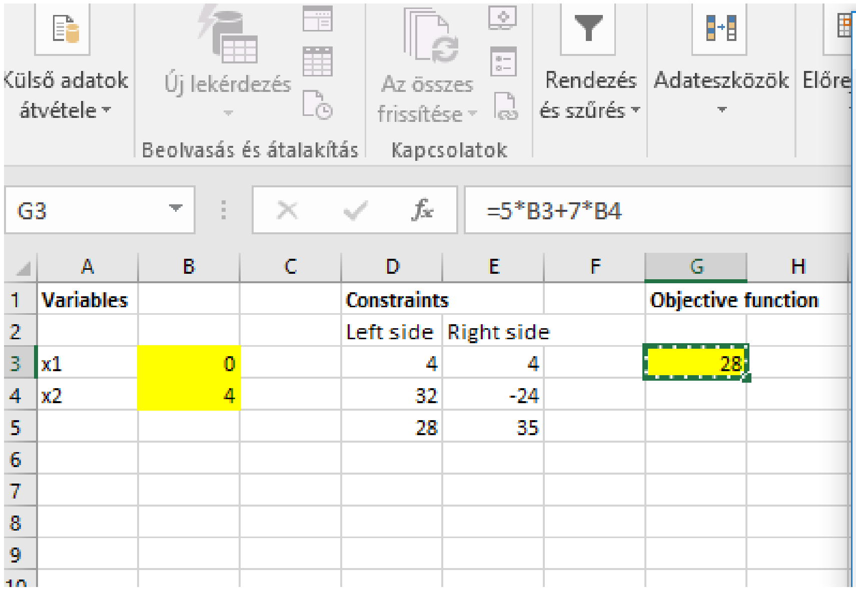
Pivot variable: 1st row 2nd column (value 1)

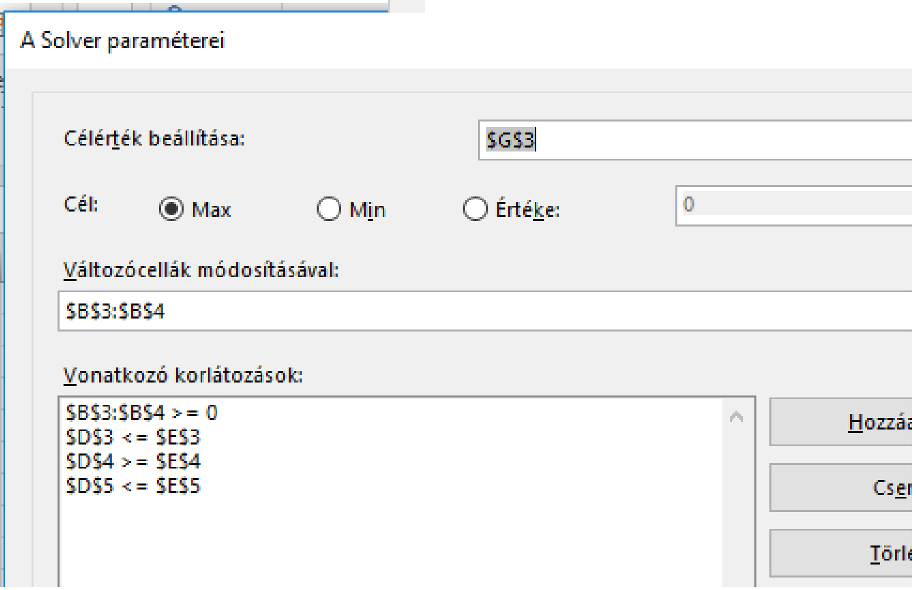
Basic variables: x_2, s_2, s_3, Z

Non-basic variables: x_1, s_1

Final solution: x_1=0; x_2=4; Z=28







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